An approach to evaporating stars and black holes

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An approach to evaporating stars and black holes

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Abstract. Many models in which the object under study loses all its mass have appeared in the literature. No doubt, the most eminent examples are the models for evaporating black holes and for evaporating stars. Here we describe a semiclassical study of these evaporating models centered on the evaporating event itself. We pay special attention to the evaporating models as a means of avoiding singularities during the collapse. In case of any pre-existing non-spacelike curvature singularity, we show that these models tend to evaporate it.

1. Introduction

The possibility that an object could radiate away its entire mass was, apparently, first considered by H. Bondi [1] as a means of avoiding the formation of singularities during the collapse of a star. The first model for an evaporating star appeared in an article by Demianski and Lasota [2]. However, it was shown later that this model in fact generates an instantaneous naked curvature singularity at the evaporating event. In this way, it became clear that the evaporating models could also be used to produce counterexamples for the Penrose's *cosmic censorship conjecture*. While many counterexamples of this type appeared later (see, for example, [3] and references therein) the original idea of avoiding the formation of singularities using evaporating models was further pursued by some authors whether in the form of evaporating stars (see [4] and references therein) or in the form of *incipient black holes* (see [5] and references therein). Of course, we cannot forget that at the same time another type of evaporating object, proposed by S. W. Hawking, was being developed: The evaporating black hole.

Our goal here is to study the evaporating models as a whole. In order to get these models we will match a general interior solution with the exterior radiating Vaidya's solution, in such a way that mass will be lost in the form of outgoing incoherent null radiation.

2. The exterior an interior solutions

Locally, Vaidya's metric [6] can be described in radiative coordinates as

$$ds_V^2 = -\bar{\chi} d\bar{u}^2 - 2\, d\bar{u}\, d\bar{R} + \bar{R}^2 \left(d\bar{\theta}^2 + \sin^2\bar{\theta} d\bar{\phi}^2\right) \tag{1}$$

where $\bar{\chi} = 1 - 2\frac{\bar{m}(\bar{u})}{\bar{R}}$ and the *mass function* $\bar{m}$ [7] depends only on $\bar{u}$.

The energy-momentum tensor for the metric (1) is of pure radiation type

$$\bar{T}_{\mu\nu} = -\frac{2}{\bar{R}^2}\frac{d\bar{m}(\bar{u})}{d\bar{u}} l_\mu l_\nu, \qquad l_\mu dx^\mu = -d\bar{u} \qquad l_\mu l^\mu = 0\,. \tag{2}$$

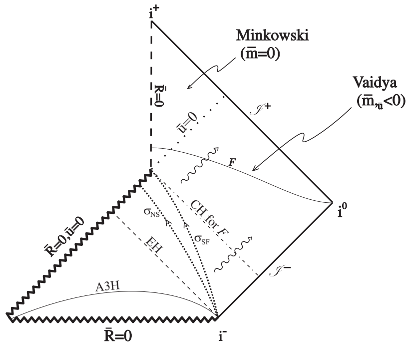

Figure 1. The Penrose diagram for Vaidya's spacetime when $\bar{m}(\bar{u})$ vanishes from some instant $\bar{u} = 0$ on and $-1/16 \leq \Upsilon \leq 0$. Note that a null singularity $\bar{u} = \bar{R} = 0$ appears.

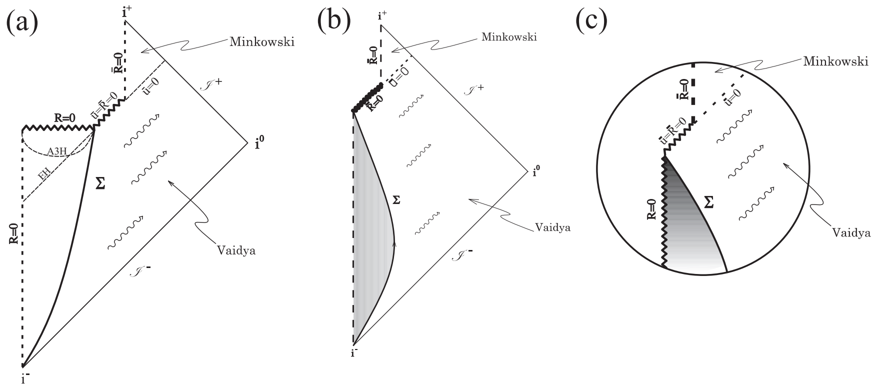

Figure 2. Forbidden possibilities

We arbitrarily set the evaporation event at $\bar{u} = 0$: $\bar{m}(\bar{u} = 0) = 0$. For $\bar{u} \geq 0$ we extend Vaidya's metric with Minkowski's spacetime, i.e. $\bar{m}(\bar{u} \geq 0) = 0$. (See [4]).

It can be shown [3] that the exterior of an evaporating object (see figure 1) must satisfy:

$$-\frac{1}{16} \leq \Upsilon \equiv \lim_{\bar{u}\to 0^-} \frac{\bar{m}(\bar{u})}{\bar{u}} \leq 0. \tag{3}$$

For the interior we will use a general four-dimensional spherically symmetric space-time $\mathcal{V}$, so that its line-element can be expressed in radiative coordinates $\{x^\mu\} = \{u, R, \theta, \varphi\}$ ($\mu = 0, 1, 2, 3$) as

$$ds^2 = -e^{4\beta}\chi du^2 + 2\varepsilon e^{2\beta}\ du\ dR + R^2\ d\Omega^2, \tag{4}$$

where $\chi \equiv 1 - 2m/R$, $\varepsilon^2 = 1$, β and m depend on $\{u, R\}$, and $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\varphi^2$. It is easily checked that $m(u, R)$ is the *mass function* [7].

Our local study is carried out in $\mathcal{U} \equiv \{(u, R)|\ \delta u \leq u \leq 0,\ 0 \leq R \leq \delta R\}$ [3]. According to the values of $m(u, R)$ and $\beta(u, R)$ and their partial derivatives computed at $u = R = 0$ we can classify $\mathcal{U}$ with a fixed causal characterization of $R = 0$ in three groups that have to be studied separately [3]: regular $\mathcal{U}$s, $\mathcal{U}$s with a non-spacelike singularity and $\mathcal{U}$s with a spacelike singularity.

3. Constructing specific evaporating models

If we write down the matching conditions for the interior and the exterior spacetimes and impose that the matched model should collapse and evaporate, we obtain the following results [3]:

Proposition 3.1 *A collapsing star endowed with an interior C^2 mass function cannot develop an exterior light-like singularity at the total evaporation event. (See figure 2).*

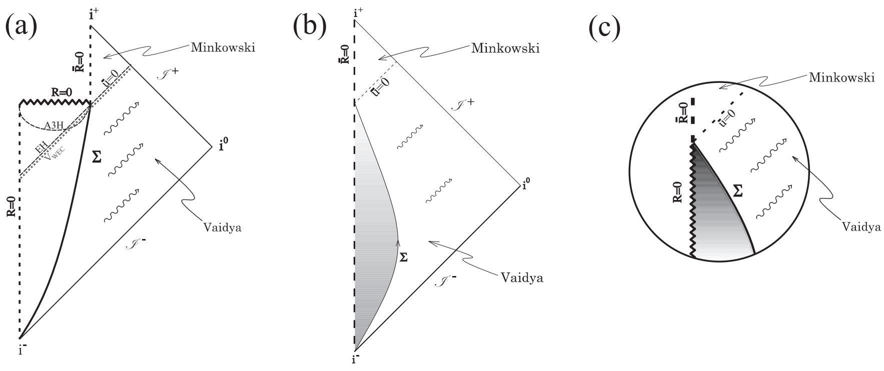

Figure 3. Different allowed evaporating models.

Proposition 3.2 *All conditions for an evaporating collapsing model with a regular interior or with an interior possessing a non-spacelike singularity will be fulfilled if, and only if, the interior satisfies* $m(u < 0, R) \overset{\Sigma_{SF}}{>} 0$ *in* $\mathcal{U} - \{u = 0, R = 0\}$ *and*

$$1 \leq \lim_{\lambda \to 0^-} \frac{m_{,R}}{R\beta_{,R}} < 2 \qquad \textit{or} \qquad \lim_{\lambda \to 0^-} \frac{2R\beta_{,R} - m_{,R} + 2m_{,u}}{2(R\beta_{,R} - m_{,R})} < 0, \tag{5}$$

for $\varepsilon = -1, +1$*, respectively. (See figures 3b and 3c).*

4. Discussion and conclusions

Our aim was to study the evaporating models as a whole. Since the physics of some evaporating models at late stages can be fairly unknown, we did not restrict the energy-momentum tensor in the interior of the model allowing classical or quantum fields.

We have shown (proposition 3.2) that a collapsing stellar model with a regular interior will evaporate if, and only if, the mass function on the matching hypersurface is non-negative and condition (5) is satisfied. In this way, the complete matched model must be singularity free. (See figure 3b). The reader can find a complete family of such models in our article [4]. Next, we have extended our study to stellar models in which an inner non-spacelike singularity (and, therefore, naked) has already formed during the collapse. In this case, we have shown that the evaporation process tends to eliminate any preexisting naked singularity. Figure 3c illustrates this situation around the evaporation event. Finally, we have treated the case in which an inner spacelike singularity preexists. These models violate the weak energy conditions and they all posses an instantaneous naked singularity that does not extend to the exterior of the collapsing object. We have illustrated this type of evaporating black holes in figure 3a.

Acknowledgements

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